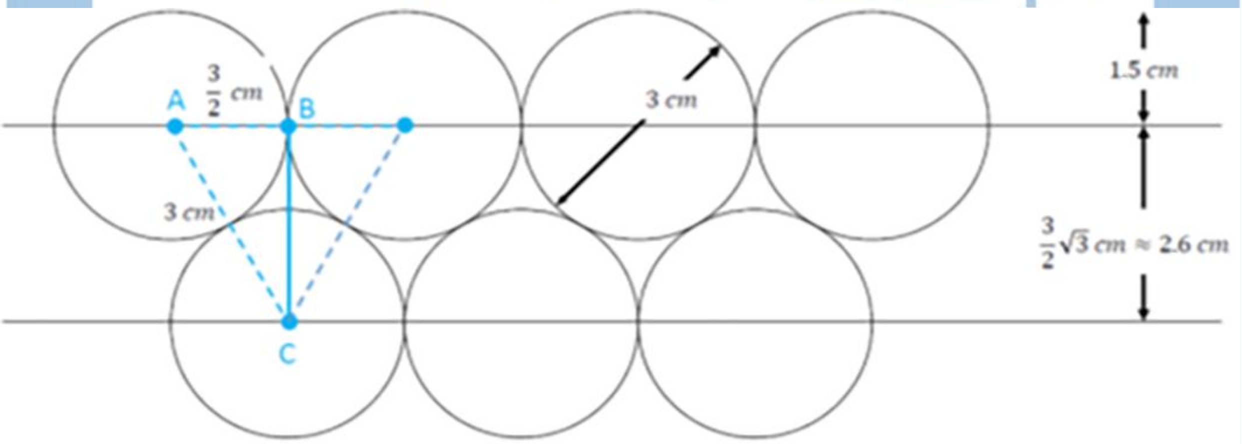
Maximum number of jacks that can be placed on a Pétanque piste $(4m \cdot 4m)$

The playing field is $400\ cm$ long and $400\ cm$ wide, and the jack has a diameter of $3\ cm$. The number of jacks that fit side by side in width, i.e., in a row, is therefore [1] $A_B = \left\lfloor \frac{400\ cm}{3\ cm} \right\rfloor = 133$.

Now we are looking for the number of jacks that can be placed one below the other over a length of $400\ cm$. The following image will help with the search:



In the densest sphere packing, the centers of two jacks lying in a row form an equilateral triangle with the center of the jack lying below them in a row, touching both of them, with a side length of $3\ cm$. The distance $\overline{BC}$ between the rows below is then (Pythagorean theorem): $\overline{BC} = \sqrt{\overline{AC}^2 - \overline{AB}^2} = \sqrt{9 - \frac{9}{4}}\ cm = \frac{3}{2}\sqrt{3} \approx 2.6\ cm$. The number of jacks that match each other in length along the length of the playing field is therefore: $A_L = \left\lfloor \frac{400\ cm}{\frac{3}{2}\sqrt{3}\ cm} \right\rfloor = 153$.

Naively speaking, finding means $A_{btw} = A_B \cdot A_L = 20{,}349$ on the playing field.

However, this is an overly optimistic interim result, as the following consideration shows:
If the left edge of the first jack in the first row marks width $0\ cm$ the right edge of the last jack in the first row falls on width $133 \cdot 3\ cm = 399\ cm$. The jacks in the second row are spaced apart from those in the first row. The second row therefore begins at width $1.5\ cm$ and ends at width $(399 + 1.5)cm = 400.5\ cm$, i.e., outside the piste. To ensure that all the jacks are **in** the playing field, the last jack must be removed from each horizontal row with an even number. That's a total of $\frac{152}{2} = 76$ jacks. (The length of the playing field is $400\ cm$.) Although it must be taken into account that the distance between the centers of the first row and the top of the playing field and the distance between the centers of the last row and the bottom of the playing field are not, as tacitly assumed above, $\frac{3}{4}\sqrt{3}\ cm$,but $1.5\ cm$, hence 153 rows fit into the playing field. When placed one below the other, their total length is $\left(153 \cdot \frac{3}{2}\sqrt{3} + 2 \cdot \left(1{,}5 - \frac{3}{4}\sqrt{3}\right)\right)\ cm \approx 397{,}91\ cm$.) This brings the total number of jacks that can fit in a $(4m \cdot 4m)$ playing area in this arrangement to a maximum of

$$A_{hor} = 20{,}349 - 76 = 20{,}273.$$

[1] The bracket $\lfloor \cdot \rfloor$ means that the result of the calculation contained therein must be rounded down to the next smaller whole number.

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Now we need to look at the alternative solution, i.e. first place the jacks lengthwise, i.e. vertically, then fill in the width:

With a length of $400\ cm$ and a jack diameter of $3\ cm$

$$A_L = \left\lfloor \frac{400\ cm}{3\ cm} \right\rfloor = 133$$

jacks fit underneath each other.

Now we are looking for the number of jacks to be placed in width. Since the same symmetry applies as in the image, the distance between the vertical rows is also $\frac{3}{2}\sqrt{3} \approx 2.6\ cm$. This means that

$$A_B = \left\lfloor \frac{400\ cm}{\frac{3}{2}\sqrt{3}\ cm} \right\rfloor = 153$$

jacks can be accommodated in a space $400\ cm\ wide$. Naively, 20,349 jacks could be accommodated on the playing field. Now comes the correction again: in each even vertical row, not 133 but only 132 jacks can be placed. his means that the total number of jacks that can be placed is now

$$A_{ver} = 20349 - 76 = 20{,}273 = A_{hor} = 20{,}273.$$

Now all that remains is to check whether the 153 vertical rows fit into the 4-meter-wide playing field, given that the leftmost and rightmost vertical rows are each 1.5 cm away from the edge of the playing field.
That is the case because it is $\left(153 \cdot \frac{3}{2}\sqrt{3} + 2 \cdot \left(1.5 - \frac{3}{4}\sqrt{3}\right)\right)\ cm \approx 397.91\ cm.$

A_{ver} and A_{hor} are equal in this case.

Accordingly, the solution here is: $A_{max} = A_{hor} = A_{ver} = 20{,}273$

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